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ॐ(OM) theory in diverse dimensions

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Abstract: Open string theories can be decoupled from closed strings and gravity by scaling to the critical electric field. We propose dual descriptions for the strong coupling limit of these NCOS (Non-Commutative Open String) theories in six or fewer spacetime dimensions. In particular, we conjecture that the 5-dimensional NCOS theory at strong coupling, is a theory of light Open Membranes (OM=ॐ), decoupled from gravity, on an M5-brane with a near-critical three-form field strength. The relation of ॐ theory to NCOS theories resembles that of M-theory to type-II closed string theories. In two dimensions we conjecture that supersymmetric U(n) gauge theory with a unit of electric flux is dual to the NCOS theory with string coupling $1/n$. A construction based on NS5-branes leads to new theories in six dimensions generalising the little string theory. A web of dualities relates all the above theories when they are compactified on tori.

Keywords: Field Theories in Higher Dimensions, M-Theory, p-branes, Non-Commutative Geometry.

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Contents

1. Introduction

It has been a long-held belief that open string theories always require closed strings for consistency at the quantum level, due to the appearance of poles in one-loop open string scattering amplitudes [1]. This belief has recently been questioned. Weakly-coupled theories of open strings on D-branes were constructed by scaling to a critical electric field, and S-duality was used to argue that they decouple from closed strings [2, 3]. The decoupling was verified, for two through 6-dimensional branes (IR problems may appear for higher dimensions), by the absence of closed string poles in non-planar loop diagrams [3]. These simplified string theories thus permit the investigation of mysterious stringy phenomena without the complications of gravity and consequent loss of a fixed background geometry. As the name NCOS theory (Non-Commutative Open String) indicates, they exhibit non-commutativity of space and time coordinates (spacetime noncommutativity was also considered in [4]–[8]. The corresponding supergravity solutions are studied in [9, 10]). In five and six dimensions they also provide a non-gravitational ultraviolet completion of Yang-Mills theory.

We expect that these theories are part of a web of theories related by duality and compactification. In this paper we explore a piece of this web by seeking strong coupling duals for all the NCOS theories. In four dimensions it was already argued in [3] (see also [5]) that the NCOS theory is dual to spatially non-commutative, maximally supersymmetric Yang-Mills field theory. In five dimensions we conjecture that the strongly coupled NCOS theory consists of M5-branes with a near-critical three-form field strength.

The M5-brane is the boundary of fluctuating open membranes, much as D-branes are the boundaries of fluctuating open strings [11, 12]. Near criticality these open membranes become nearly tensionless. This theory — ॐ (OM)[1] theory — is described by the gravitationally decoupled dynamics of the light open membranes.[2]

The M5-brane near a critical three-form field, and its compactifications were considered in [14], however we do not consider the rank four case of that paper. Decoupled theories with constant C have also been explored by various authors including [9], [14]–[17]. It has been conjectured that just as non-zero B is related to noncommutativity, non-zero C might be related to nonassociativity. However, it is not clear how to make this conjecture precise.

We go on to define a large class of new 6-dimensional non-gravitational theories with light open D-branes among their excitations. Specifically, these are scaling theories on NS5-branes with near critical RR gauge fields of different ranks. This results in the presence of corresponding light branes in the spectrum. These are part of the ॐ web of theories, being related by various dualities on circle compactification.

The 2-dimensional NCOS theory (see [18, 19, 20]) has the unique feature that the open string coupling is quantized and bounded, $G_o^2 = 1/n \leq 1$; thus there is no strong coupling limit. However we argue that the 2-dimensional NCOS theory at weak coupling (large n) is dual to strongly coupled 2-dimensional U(n) gauge theory with discrete electric flux. We argue that the strong coupling limit of the 3-dimensional NCOS theory is the SO(8) invariant M2-brane worldvolume field theory.

[1]ॐ (OM): that which captures the underlying nature of reality [13], or Open Membrane, according to taste.

[2]Just as for M-theory, we will interchangeably use the term ॐ theory also for the whole web of non-gravitational theories related via compactifications and dualities to the theory with light open membranes.

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Note added to revised edition. Related independent work has appeared in [21]. Several papers analysing the properties of ॐ theory have also appeared since the original version.

2. OM theory

In this section we will consider the theory of an M5-brane in the presence of a near critical electric H_{012} field. We will find that in the limit $\frac{H_{\rm crit}-H}{H_{\rm crit}} \to 0$, the tension of open membranes stretched spatially in the $1,2$ directions is infinitely below the Planck scale. It is thus possible to define a theory of light fluctuating open membranes propagating on the M5-brane, decoupled from gravity. Like the $(0,2)$ theory, ॐ theory has no dimensionless parameters, and so is unique and strongly coupled. In fact ॐ theory reduces to the $(0,2)$ field theory at low energies.

Consider M-theory in the presence of N coincident M5-branes with a background worldvolume three-form field strength

$$H_{012} = M_{\rm p}^3 \tanh\beta\,, \tag{2.1}$$

and an asymptotic metric

$$g_{\mu\nu} = \eta_{\mu\nu}\,, \qquad g_{ij} = f^2\delta_{ij}\,, \qquad g_{MN} = h^2\delta_{MN}\,, \tag{2.2}$$

with $\mu,\nu = 0,1,2$, $i,j = 3,4,5$, $M,N = 6,7,8,9,11$. f and h are constants introduced for later convenience. The non-linear self-duality constraints [22] then determine the other components of H as

$$H_{345} = -f^3 M_{\rm p}^3 \sinh\beta\,. \tag{2.3}$$

The effective tension of a membrane (proper mass per unit proper area) stretched spatially in the $1,2$ directions is

$$\frac{1}{4\pi^2}\left(M_{\rm p}^3 - \epsilon^{012}H_{012}\right) = \frac{M_{\rm p}^3}{4\pi^2 e^\beta\cosh\beta} \equiv \frac{1}{4\pi^2}M_{\rm eff}^3\,. \tag{2.4}$$

$M_{\rm p}$ is the gravitational scale while $M_{\rm eff}$ sets the scale for the proper energies of fluctuations of these open membranes. As H_{012} is scaled to its critical value (i.e. β is taken to ∞), $M_{\rm p}/M_{\rm eff} \sim e^{2\beta/3}\!\to\!\infty$, and the fluctuating open membranes decouple from gravity! In order to focus on these light modes we take the limit as $M_{\rm p} \sim e^{2\beta/3}\!\to\!\infty$, $M_{\rm eff}$ fixed.

We have judiciously chosen μ,ν coordinates in (2.2) so that the energy per unit coordinate area of a membrane aligned along the 012 direction is finite. This condition does not fix the i,j coordinate system or equivalently a choice of f in (2.2). We

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fix this by demanding that a membrane along e.g. 034 has finite energy per unit coordinate area. Since the energy per unit proper area of such a membrane (as measured by g_{ij}) diverges like $M_{\rm p}^3$ in the limit, this requires a small f:

$$\epsilon_{034} M_{\rm p}^3 = \text{finite} \Longrightarrow f^2 \propto \frac{M_{\rm eff}^3}{M_{\rm p}^3}\,. \tag{2.5}$$

For later convenience we make the specific choice[3]

$$f^2 = 2\frac{M_{\rm eff}^3}{M_{\rm p}^3}\,. \tag{2.6}$$

It is also convenient to choose

$$h^2 = f^2 = 2\frac{M_{\rm eff}^3}{M_{\rm p}^3}\,. \tag{2.7}$$

Below we will argue that, with this choice of transverse coordinates, the dimension two scalar operators Φ^M (normalized to have the usual kinetic term) representing the transverse fluctuations of the 5-brane in the low energy field theory limit of OM theory, are related to the geomertrical position of the 5-brane by a factor of $M_{\rm eff}^3$; $\Phi^M \sim M_{\rm eff}^3 X^M$.

In summary, we consider N M5-branes in the ॐ limit

$$\beta \longrightarrow \infty\,, \qquad M_{\rm eff}\ \text{fixed}\,, \quad (\mu,\nu = 0,1,2,\ i,j = 3,4,5,\ M,N = 6,7,8,9,11.)$$

The resultant ॐ theory contains fluctuating open membranes of proper energy $\mathcal{O}(M_{\rm eff})$, decoupled from gravity. Note that ॐ theory has no dimensionless parameters. We will argue below that ॐ theory, upon compactification in the 2 direction, reduces to the $(4+1)$-dimensional NCOS theory, and therefore is not a trivial theory, even in the case $N = 1$. Thus, for simplicity, we will concentrate on the theory of a single 5-brane through the rest of this paper, although our considerations may be generalized.

The ॐ limit
$M_{\rm p}^3 = \frac{M_{\rm eff}^3}{2} e^{2\beta}$
$H_{012} = M_{\rm p}^3 \tanh\beta$
$g_{\mu\nu} = \eta_{\mu\nu}$
$g_{ij} = \frac{2M_{\rm eff}^3}{M_{\rm p}^3}\delta_{ij}$
$g_{MN} = \frac{2M_{\rm eff}^3}{M_{\rm p}^3}\delta_{MN}$

Table 1

3. Review of the NCOS limit

In subsequent sections we will study OM theory compactified on various circles. In particular, we will find a relationship between OM theory and the $(4+1)$-dimensional NCOS theory. In this section we review the NCOS limit in coordinates convenient for present purposes.[4] We will also discuss the T-duality of NCOS theories.

[3] It is possible that the factor of 2 can be motivated from isotropy of the "open membrane metric" [16] but we shall not do so here.

[4] Our conventions and coordinates here differ from those employed in [3], which were chosen to elucidate the relation to the S-dual field theory.

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Consider a Dp-brane with a near critical electric field in the $0,1 \equiv \mu,\nu$ direction and closed string coupling denoted by $g_{\rm str}$. The closed string metric and electric field can be chosen as

$$g_{\mu\nu} = \eta_{\mu\nu}\,, \qquad g_{ij} = \epsilon\delta_{ij}\,, \qquad g_{MN} = \epsilon\delta_{MN}\,, \qquad 2\pi\alpha'\epsilon^{01}F_{01} = 1 - \frac{\epsilon}{2}\,, \tag{3.1}$$

with $\epsilon \ll 1$, $i,j = 2,3,\ldots,p$ the non-electric directions on the brane, and $M,N = p{+}1,\ldots,9$ the directions transverse to the brane. Using the formulae in [23, 24, 14] it is easy to determine the open string metric, noncommutativity parameter and string coupling G_o^2 corresponding to the closed string moduli of (3.1),

$$G_{\mu\nu} = \epsilon\eta_{\mu\nu}\,, \qquad G_{ij} = \epsilon\delta_{ij}\,, \qquad \Theta^{\mu\nu} = \frac{2\pi\alpha'}{\epsilon}\epsilon^{\mu\nu}\,, \qquad G_o^2 = g_{\rm str}\sqrt{\epsilon}\,. \tag{3.2}$$

The effective tension (energy per unit length) of a string stretched in the 1 direction is

$$\frac{1}{2\pi}\left(\frac{1}{\alpha'} - 2\pi\epsilon^{01}F_{01}\right) = \frac{\epsilon}{4\pi\alpha'} \equiv \frac{1}{4\pi\alpha'_{\rm eff}}\,. \tag{3.3}$$

α' sets the scale of closed string oscillators, and $\alpha'_{\rm eff}$ the scale for the energy of oscillating open strings. As the electric field is scaled to its critical value, $\alpha'/\alpha'_{\rm eff} = \epsilon{\to}0$, and the oscillating open strings decouple from gravity. We take $\alpha'_{\rm eff}$ fixed as $\epsilon{\to}0$, so that $\alpha' \propto \epsilon{\to}0$. Open string oscillator states obey the mass shell condition

$$p_A\eta^{AB}p_B = \frac{N}{\alpha'_{\rm eff}} \tag{3.4}$$

and so have proper energy $p_0^2 = \mathcal{O}(1/\alpha'_{\rm eff})$ as expected. The part of the string sigma model involving transverse coordinates is

$$S = \frac{1}{4\pi\alpha'}\int g_{MN}\partial X^M\bar\partial X^N = \frac{1}{4\pi\alpha'_{\rm eff}}\int \delta_{MN}\partial X^M\bar\partial X^N\,. \tag{3.5}$$

Thus correlation functions of the X^M fields are finite as $\alpha'{\to}0$, and the dimension one scalar fields ϕ^M (normalized to have standard kinetic term) in the low energy gauge theory on the NCOS brane worldvolume, are related to the coordinates X^M by a factor of $1/\alpha'_{\rm eff}$; $\phi^M \sim X^M/\alpha'_{\rm eff}$. Finally, we scale $g_{\rm str}$ to keep $G_o^2 = g_{\rm str}\sqrt{\alpha'/\alpha'_{\rm eff}}$ fixed as α' is taken to zero. This limit (the NCOS limit) results in a one parameter family of interacting open string theories, (NCOS theories), labelled by their coupling constant G_o and decoupled from gravity.

At low energies, the NCOS theory reduces to

$$\begin{aligned} S &= \frac{1}{4(2\pi)^{p-2}G_o^2\alpha'^{\frac{p-3}{2}}}\int d^{p+1}x\sqrt{-G}G^{AM}G^{BN}\hat F_{AB}\hat F_{MN} \\ &= \frac{1}{4(2\pi)^{p-2}G_o^2\alpha'^{\frac{p-3}{2}}_{\rm eff}}\int d^5x\eta^{AM}\eta^{BN}\hat F_{AB}\hat F_{MN} \end{aligned} \tag{3.6}$$

i.e. it reduces to Yang Mills theory with $g^2_{\rm YM} = (2\pi)^{p-2}G_o^2\alpha'^{\frac{p-3}{2}}_{\rm eff}$.

3.1 T duals of NCOS theories

The NCOS limit
$g_{\mu\nu} = \eta_{\mu\nu}$
$g_{ij} = \frac{\alpha'}{\alpha'_{\rm eff}}\delta_{ij}$
$g_{MN} = \frac{\alpha'}{\alpha'_{\rm eff}}\delta_{MN}$
$2\pi\epsilon^{01}\alpha' F_{01} = 1 - \frac{\alpha'}{2\alpha'_{\rm eff}}$
$g_{\rm str} = G_o^2\sqrt{\frac{\alpha'_{\rm eff}}{\alpha'}}$
$G^{AB} = \frac{\alpha'_{\rm eff}}{\alpha'}\eta^{AB}$
$\Theta^{\mu\nu} = 2\pi\alpha'_{\rm eff}\epsilon^{\mu\nu}$

Table 2

In this section we review the action of T-duality on the NCOS theories for later use.[5] Consider a $(p+1)$-dimensional brane in the NCOS limit of table 2, wrapped on a circle of coordinate radius R in the p-th spatial direction. Performing a T-duality in the p-th direction yields a $(p-1)+1$ dimensional brane, at a point on the now transverse p-th circle, whose coordinate radius is given by

$$\tilde{R} = \frac{\alpha'_{\rm eff}}{\alpha'} \times \frac{\alpha'}{R} = \frac{\alpha'_{\rm eff}}{R}\,. \tag{3.7}$$

The asymptotic value of the string coupling after T-duality, $g'_{\rm str}$, is given by

$$g'_{\rm str} = g_{\rm str}\frac{\sqrt{\alpha'}}{\sqrt{\frac{\alpha'}{\alpha'_{\rm eff}}}R} = g_{\rm str}\frac{\sqrt{\alpha'_{\rm eff}}}{R} = \frac{G_o^2\sqrt{\alpha'_{\rm eff}}}{R}\sqrt{\frac{\alpha'_{\rm eff}}{\alpha'}}\,. \tag{3.8}$$

The asymptotic values of the metric, and the worldvolume electric field are unchanged by the T-duality, and so may be read off from table 2.

In conclusion, the $(p+1)$-dimensional NCOS theory with scale $\alpha'_{\rm eff}$ and coupling G_o, wrapped on a circle of coordinate length R in a non-electric direction, is T-dual to a $(p-1)+1$ dimensional NCOS theory with one compact transverse scalar. The $(p-1)+1$ dimensional NCOS theory has scale $\alpha'_{\rm eff}$ and coupling $G_o'^2 = G_o^2\sqrt{\alpha'_{\rm eff}}/R$. The coordinate radius of compactification of the transverse scalar is $\alpha'_{\rm eff}/R$.[6] Notice that the NCOS radius and coupling transform under T-duality exactly as the analogeous closed string quantities transform under the usual closed string T-duality, with $\alpha'_{\rm eff}$ playing the role of α'.

4. Compactification of OM theory on an electric circle

Consider ॐ theory compactified on a spatial circle of proper (and coordinate) radius R in one of the "electric" spatial directions (the direction x^2 for definiteness). Since ॐ theory reduces to the $(0,2)$ theory at energies well below $M_{\rm eff}$, the low energy dynamics of the compactified theory is governed by $(4+1)$-dimensional Yang Mills with $g_{\rm YM}^2 \sim R$. At higher energies light open membranes wrap the compactification circle to form the light open strings decoupled from gravity. It is natural to guess that this theory is the 5-dimensional NCOS theory with effective string tension $\frac{1}{4\pi\alpha'_{\rm eff}} \sim M_{\rm eff}^3 R$ and open string coupling $G_o^2 \sim g_{\rm YM}^2/\sqrt{\alpha'_{\rm eff}} \sim (RM_{\rm eff})^{3/2}$. In this section we will verify that this is indeed the case.

[5] This discussion has independently appeared in [25].

[6] The dimension one field ϕ^p (normalized to have the usual kinetic term) in the low energy gauge theory is compact with radius $\sim 1/R$.

ॐ theory compactified on a spatial circle (say in the 2 direction) of proper radius R, may be obtained as follows. Consider M-theory on $S^1 \times R^{10}$ (the S^1 is in the 2 direction) with M5-branes wrapping the circle. Scale all bulk moduli as in section 2; in particular

$$\begin{aligned}
\epsilon^{012} H_{012} &= M_{\rm p}^3 - M_{\rm eff}^3\,, \qquad g_{\mu\nu} = \eta_{\mu\nu} \quad (\mu,\nu = 0,1,2)\,,\\
g_{ij} &= \frac{2M_{\rm eff}^3}{M_{\rm p}^3}\delta_{ij} \quad (i,j = 3,4,5)\,,\\
g_{MN} &= \frac{2M_{\rm eff}^3}{M_{\rm p}^3}\delta_{MN} \quad (M,N = \text{transverse})\,, \quad M_{\rm p} \longrightarrow \infty\,, \quad M_{\rm eff} \text{ fixed}\,.
\end{aligned} \tag{4.1}$$

The dictionary between M-theory and IIA implies that this system is equivalent to a D4-branes in IIA theory with

$$\begin{aligned}
&\frac{1}{\alpha'} = RM_{\rm p}^3\,, \qquad g_{\rm str} = (RM_{\rm p})^{3/2}\,,\\
&g_{\mu\nu} = \eta_{\mu\nu} \quad (\mu,\nu = 0,1)\,, \qquad g_{ij} = \frac{2M_{\rm eff}^3}{M_{\rm p}^3}\delta_{ij} \quad (i,j = 3,4,5)\,,\\
&g_{MN} = \frac{2M_{\rm eff}^3}{M_{\rm p}^3}\delta_{MN} \quad (M,N = \text{transverse})\,, \qquad 2\pi\alpha' F_{01} = \alpha' R H_{012} = 1 - \frac{M_{\rm eff}^3}{M_{\rm p}^3}\,.
\end{aligned} \tag{4.2}$$

In the limit $M_{\rm p} \to \infty$, comparing with table 2, we find ourselves in the NCOS limit, $\alpha' \to 0$ with fixed effective open string tension and open string coupling

$$\frac{1}{2\alpha'_{\rm eff}} = \frac{M_{\rm eff}^3}{M_{\rm p}^3\alpha'} = M_{\rm eff}^3 R\,, \qquad G_o^2 = g_{\rm str}\sqrt{\frac{2M_{\rm eff}^3}{M_{\rm p}^3}} = \sqrt{2}(RM_{\rm eff})^{3/2}\,. \tag{4.3}$$

Thus the $(4+1)$-dimensional NCOS theory with scale $\alpha'_{\rm eff}$ and coupling G_o^2 may be identified with ॐ theory with scale $M_{\rm eff}$, compactified on an electric circle of radius R with

$$M_{\rm eff} = \frac{2^{-1/3}}{\sqrt{\alpha'_{\rm eff}}G_o^{2/3}}\,, \qquad R = G_o^2\sqrt{\alpha'_{\rm eff}}\,. \tag{4.4}$$

The relation between ॐ theory and the 5-dimensional NCOS theory is reminiscent of the relationship between M-theory and IIA string theory. Notice that the radius of the compactification circle in units of $1/M_{\rm eff}$ is equal to $G_o^{4/3}$. Thus, at strong NCOS coupling Kalutza Klein modes are much lighter than $M_{\rm eff}$ and the theory is effectively 6 dimensional.

As argued in the previous section, the dimension one scalar field ϕ^M (normalized to have unit kinetic term) on the worldvolume of the NCOS brane is related to transverse coordinate position, by $\phi^M \sim X^M/\alpha'_{\rm eff}$. However, the corresponding dimension 2 field Φ^M on the OM worldvolume, is related to ϕ^M by $\Phi^M \sim \phi^M/R$. Combining these formulae, and using (4.4) we conclude that $\Phi^M \sim M_{\rm eff}^3 X^M$, as asserted in section 2.

5. Compactification of OM theory on a magnetic circle

Again consider ॐ theory compactified on a spatial circle, this time of *coordinate* radius L (proper radius $\sqrt{2M_{\rm eff}^3/M_{\rm p}^3 L}$) in one of the "magnetic" spatial directions (the direction x^3 for definiteness). As in the previous subsection, the compactified theory at low energies is $(4+1)$-dimensional SYM with gauge coupling $g_{\rm YM}^2 \sim L$. Indeed, we will see below that the effective $(4+1)$-dimensional description of this theory is $(4+1)$-dimensional non-commutative SYM. Since non-commutative SYM is nonrenormalizable in five dimensions, the theory does not have a complete $(4+1)$-dimensional description. ॐ theory on a circle provides a completion of $(4+1)$-dimensional non-commutative SYM.

Proceeding as in the previous section, we find that the compactified theory may equivalently be described as a D4-brane in IIA theory with parameters

$$\begin{aligned} \alpha' &= \frac{1}{L\sqrt{2M_{\rm eff}^3 M_{\rm p}^3}}\,, \qquad g_{\rm str} = \left(\frac{2L^2 M_{\rm eff}^3}{M_{\rm p}}\right)^{3/4}, \qquad g_{\mu\nu} = \eta_{\mu\nu} \quad (\mu,\nu = 0,1,2)\,, \\ g_{ij} &= 2\frac{M_{\rm eff}^3}{M_{\rm p}^3}\delta_{ij} \quad (i,j = 4,5)\,, \qquad F_{45} = \frac{LM_{\rm eff}^3}{\pi}\,. \end{aligned} \tag{5.1}$$

As in [14], (the parameter ϵ of [14] may be identified with $e^{-2\beta}$ in (5.1)) the decoupled field theory on the D4-brane is maximally supersymmetric U(1) non-commutative Yang Mills (NCYM) with open string metric

$$G_{\mu\nu} = \eta_{\mu\nu}\,, \qquad G_{ij} = (2\pi\alpha' F_{45})^2 \frac{M_{\rm p}^3}{2M_{\rm eff}^3}\delta_{ij} = \delta_{ij} \tag{5.2}$$

and noncommutativity

$$\Theta^{ij} = \frac{\epsilon^{ij}}{F_{45}} = \frac{\pi\epsilon^{ij}}{LM_{\rm eff}^3}\,. \tag{5.3}$$

At low energies, the NCYM theory is governed by the lagrangian

$$\mathcal{L} = \frac{1}{4g_{\rm YM}^2}\int d^5x\sqrt{-G}G^{AM}G^{BN}\hat{F}_{AB}\hat{F}_{MN}\,, \tag{5.4}$$

where

$$g_{\rm YM}^2 = (2\pi)^2\sqrt{\alpha'}g_{\rm str}\sqrt{\frac{\det G_{ij}}{\det(2\pi\alpha' F_{ij})}} = 4\pi^2 L \tag{5.5}$$

as expected.

5.1 Compactification on an electric and a magnetic circle

Another interesting compactification which combines the two previous ones is on the circle $(x^2,x^5) \sim (x^2,x^5) + (2\pi L_2, 2\pi L_5)$. (The following discussion has also appeared

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independently in [26]. See also [8, 27, 25].) Since in the metric g the distances along x^5 are scaled to zero, the radius of the circle in the scaling limit is independent of L_5. It is given by L_2, and therefore α' and the string coupling are as in the compactification on the electric circle. The two form in the non-compact directions is determined as $F = \oint H = 2\pi L_2 H_{012} dx^0 dx^1 + 2\pi L_5 H_{345} dx^3 dx^4$.

We start by analyzing the situation in directions $0, 1$. Since the electric field scales to the critical value as in the compactification on the electric circle, the open string metric G and the noncommutativity parameter Θ in these directions are as in that problem, i.e. G scales like $1/M_p^3$ and Θ^{01} is finite.

In directions $3, 4$ the situation differs from that in the previous cases. Here g scales like $1/M_p^3$ and F is of order one. Since α' is of order $1/M_p^3$, the two terms in the denominator of $G^{-1} + \frac{1}{2\pi\alpha'}\Theta = \frac{1}{g+2\pi\alpha' F}$ are of the same order of magnitude. We conclude that the components of G in these directions are of order $1/M_p^3$ and that Θ^{34} is of order one.

Finally, there is one more non-compact direction. It is straightforward to check that the induced metric along that direction is of order $1/M_p^3$, and therefore G is also of that order.

We conclude that all the components of G are of order $1/M_p^3$ and that Θ^{01} and Θ^{34} are of order one. This is similar to the situation with the electric circle except that there is also noncommutativity in the spatial directions; i.e. Θ is of rank four.

6. Near critical NS5-brane theories

In this section we will define a series of new 6-dimensional theories, ODp (Open Dp-brane) theories,[7] labelled by an integer p where p runs from 1–5. These are decoupled theories on the world volume of the NS5-brane; the excitations of these theories include light open Dp-branes. These Dp-branes remain light, even in the decoupling limit, due to the presence of a near critical $p + 1$ form Ramond-Ramond potential (NS5-branes with background RR fields were studied by various authors including [28]).

As is well known, all even Dp-branes can end on NS5-branes in the IIA theory and all odd Dp-branes can end on NS5-branes in the IIB theory (when $p < 6$). Therefore, with an appropriate background RR field such open Dp-branes can be made very light. This is similar to the light open fundamental strings on D-branes in the NCOS theories and the light open membranes of ॐ theory, discussed in the previous sections.

Our construction of near critical NS5-brane theories can be generalized by turning on several different Ramond-Ramond fields. In these theories the light D-branes carry several charges. We will not analyze these generalized theories in detail in this paper.

[7] We have learnt of independent related work by T. Harmark (to appear).

The various near critical NS5-brane theories are distinct 6-dimensional theories but, upon compactification, all lie on the same moduli space. This is similar to the equivalence of the type-IIA and type-IIB string theories when compactified on a circle and to the equivalence of the IIA and IIB little string theories [29]. As we will argue below, ODp theories are also on the same moduli space as NCOS theories and ॐ theory.

In order to motivate our construction, consider a D5-brane in IIB theory, in the $(5+1)$-dimensional NCOS limit, as given in table 2. S-dualizing this background yields a scaling limit that defines the OD1 theory, a decoupled theory on the world volume of the IIB-NS5-brane, whose excitations are open D-strings. These are light in the decoupling limit because of a near critical background C_{01} RR potential. Compactifying this theory on tori, T-dualizing, and decompactifying the resultant theories, yields the scaling limits that define the various ODp theories. These scaling limits are summarized in table 3 below.

ODp theories
$g_{\mu\nu} = \eta_{\mu\nu}, \quad \mu,\nu = 0,1,\ldots,p$
$g_{MN} = \epsilon\delta_{MN}, \quad M,N = (p+1),\ldots,9$
$\tilde{\alpha}' = \epsilon^{1/2}\tilde{\alpha}'_{\rm eff}$
$g_{\rm str}^{(p)} = \epsilon^{\frac{3-p}{4}} G^2_{{\rm o}(p)}$
$\epsilon^{01\cdots p}C_{01\cdots p} = \frac{1}{(2\pi)^p G^2_{{\rm o}(p)} \tilde{\alpha}'^{\frac{p+1}{2}}_{\rm eff}}(\frac{1}{\epsilon} - \frac{1}{2})$
$C_{(p+1)\cdots 5} = \frac{1}{(2\pi)^{4-p} G^2_{{\rm o}(p)} \tilde{\alpha}'^{\frac{5-p}{2}}_{\rm eff}}$

Table 3

In the table above $\tilde{\alpha}'$ and $g_{\rm str}^{(p)}$ represent the closed string scale and closed string coupling, respectively. Note that M,N run over all dimensions transverse to the brane, as well as the brane directions orthogonal to the critical C-field.

Like the NCOS theories, the ODp theories are labelled by two parameters, the dimensionless $G_{{\rm o}(p)}$ and a scale $\tilde{\alpha}'_{\rm eff}$. Below we will comment on the interpretation of these parameters.

As an aside we note that in table 3 we have made use of the fact that the components of the RR potentials on the NS5-branes, which cannot be gauged away are subject to a non-linear equation similar to that of the three form on the M5-brane. To see that, start with an M5 with a large transverse circle. Then the three form is constrained by that equation. Making the circle small we can interpret the theory as type-IIA string theory and the three form is an RR potential, which is subject to the same non-linear equation. By compactifying some of the directions and using T-duality, a similar equation for the other RR background fields is easily derived. S-dualizing also leads to similar relations on the worldvolume of the D5-brane (see section 6.4 ahead).

Consider the ODp limit, with the NS5-brane wrapped on a circle in the p direction, with identification $x^p \sim x^p + 2\pi R_p$. Under T-duality in the p direction, bulk quantities transform in the usual manner. Note, in particular, that the background RR fields change rank under the T-duality transformation

$$C_{01\cdots(p-1)} = 2\pi R_p C_{01\cdots p}\,, \qquad C_{p\cdots 5} = \frac{R_p}{2\pi\tilde{\alpha}'_{\rm eff}} C_{(p+1)\cdots 5}\,, \tag{6.1}$$

and it is easily checked that the resulting scaling limit is that of the ODq (q = p − 1) theory with unchanged effective scale $\tilde{\alpha}'_{\rm eff}$, on a circle of coordinate identification $x^p \sim x^p + 2\pi\tilde{R}_p$ and dimensionless parameter $G_{o(p-1)}$ given by

$$\tilde{R}_p = \frac{\tilde{\alpha}'_{\rm eff}}{R_p}\,, \qquad G^2_{o(p-1)} = \frac{\sqrt{\tilde{\alpha}'_{\rm eff}}}{R_p} G^2_{\rm o(\it p)}\,. \tag{6.2}$$

Thus, like little string theories, ODp theories inherit the action of T-duality from the underlying string theory.

When an ODp theory is compactified on a torus whose metric is not diagonal (in the coordinate system in which the the RR fields are as given in table 3) the action of T-duality generates RR fields of different ranks. Decompactifying the T-dual torus one obtains the other 6-dimensional theories with several different RR fields which have referred to above.

6.1 $p = 0$

The OD0 limit contains NS5-branes in the presence of a near critical one-form gauge field $C_0 = \frac{1}{\epsilon G^2_{\rm o(0)}\sqrt{\tilde{\alpha}'_{\rm eff}}}(1-\epsilon/2)$, leading to light D0-branes. In contrast to ODp theories with $p > 0$ (and to NCOS and OM theories), the light excitations of the OD0 theory carry a conserved charge. Thus the OD0 may be studied in any of an infinite number of super-selection sectors, labelled by D0-brane charge.

Since we are in IIA we can lift the NS5-brane to a an M5-brane on a transverse circle with radius and Planck mass

$$R_{11} = g^{(0)}_{\rm str}\sqrt{\tilde{\alpha}'} = \epsilon G^2_{\rm o(0)}\sqrt{\tilde{\alpha}'_{\rm eff}} \equiv \epsilon R\,, \qquad M_p^3 = \frac{1}{\epsilon^{3/2} G^2_{\rm o(0)} \tilde{\alpha}'^{3/2}_{\rm eff}} \equiv \frac{1}{\epsilon^{3/2}}\tilde{M}^3_{\rm eff}\,. \tag{6.3}$$

Choosing coordinates in the 11-th direction such that ($x^{11} \sim x^{11} + 2\pi R$), the 11-dimensional metric is

$$\begin{aligned} ds^2_M &= -(dx^0)^2 + R^2_{11}\left(\frac{dx^{11}}{R} - C_0 dx^0\right)^2 + \epsilon dx^2_\perp \\ &= \epsilon^2 (dx^{11})^2 - \epsilon (dx^0)^2 - \epsilon dx^{11} dx^0 + \epsilon dx^2_\perp\,. \end{aligned} \tag{6.4}$$

Rescaling the unit of length by a factor of $\sqrt{\epsilon}$ (so that all lengths are larger, and all masses smaller, by a factor of $\sqrt{\epsilon}$) the metric, in the limit $\epsilon \to 0$, takes the simple form

$$ds^2_M = -(dx^0)^2 - dx^{11} dx^0 + dx^2_\perp\,. \tag{6.5}$$

Note that the compactified direction x^{11} is light-like. The bulk Planck scale in the new units is equal to $\tilde{M}_{\rm eff}$.

In summary, the OD0 theory with N units of D0-brane charge is a DLCQ compactification of M-theory, (with Planck scale $\tilde{M}_{\rm eff} = \frac{1}{G^{2/3}_{\rm o(0)}\tilde{\alpha}'^{1/2}_{\rm eff}}$) with N units of DLCQ momentum, in the presence of a transverse M5-brane. The periodic light-like coordinate has an identification of radius $R = G^2_{\mathrm{o}(0)}\sqrt{\tilde{\alpha}'_{\mathrm{eff}}}$.

The ODp theories for p> 1 (like NCOS theories and ॐ theory) have excitations that are open p branes. The presence of the appropriate near critical RR potential keeps these open branes light, in the decoupling limit, provided they are appropriately oriented (branes with the opposite orientation decouple). The counterpart of this statement in the OD0 theories, is the fact that that D0 number must be positive; i.e. the familiar statement that the discrete momentum around a circle must be positive in a DLCQ compactification.

6.2 $p = 1$

As pointed out above, the OD1 theory (with parameters $G_{0(1)}, \tilde{\alpha}'_{\mathrm{eff}}$) is S-dual to the $(5+1)$-dimensional NCOS theory (with parameters $G_o, \alpha'_{\mathrm{eff}}$). The relationship between NCOS parameters (defined by table 2) and ODp parameters (defined by table 3) is

$$G^2_{\mathrm{o}(1)} \equiv \frac{1}{G_o^2}\,, \qquad \tilde{\alpha}'_{\mathrm{eff}} \equiv \alpha'_{\mathrm{eff}} G_o^2\,. \tag{6.6}$$

Notice the formal analogy to the transformation of the closed string quantities α' and g_{str} under S-duality.

Note, of course, that D1-branes are exactly tensionless when C_{01} takes its critical value as $\epsilon^{01}C^{\mathrm{crit}}_{01} = \frac{1}{2\pi\epsilon G^2_{\mathrm{o}(1)}\tilde{\alpha}'_{\mathrm{eff}}} = \frac{1}{2\pi\alpha'_{\mathrm{eff}}\epsilon}$. Unsurprisingly, the effective tension of D1-branes in the OD1 limit is identical to that of NCOS strings in the S-dual $(5+1)$-dimensional NCOS theory

$$T^{(1)}_{\mathrm{eff}} = \frac{1}{4\pi\alpha'_{\mathrm{eff}}} \equiv \frac{1}{4\pi G^2_{\mathrm{o}(1)}\tilde{\alpha}'_{\mathrm{eff}}}\,. \tag{6.7}$$

At low energies the OD1 theory reduces to a $(5+1)$-dimensional gauge theory with Yang-Mills coupling $g^2_{\mathrm{YM}} = (2\pi)^3\alpha'_{\mathrm{eff}}G_o^2 = (2\pi)^3\tilde{\alpha}'_{\mathrm{eff}}$. Instantons in this gauge theory are strings (identified with fundamental strings) in the low energy limit of the OD1 theory; these strings consequently have tension $\sim 1/\tilde{\alpha}'_{\mathrm{eff}}$. This yields an interpretation for the parameter $\tilde{\alpha}'_{\mathrm{eff}}$; it sets the tension for closed little strings in OD1 theories. As the tension of a little string is unchanged under T-duality, this statement is true of all the ODp theories.

As an aside, consider the $(5+1)$-dimensional NCOS theory, in the limit $\alpha'_{\mathrm{eff}}\to 0$, $G_o\to\infty$, g^2_{YM} held fixed. $\Theta^{01} = \epsilon^{01}2\pi\alpha'_{\mathrm{eff}}\to 0$ in this limit, and so (at least naively) the NCOS theory recovers $(5+1)$-dimensional Lorentz invariance in this limit. Open string oscillator states are infinitely massive, and decouple in this limit. Thus we are left with an (apparently) Lorentz invariant 6-dimensional theory, whose low energy limit is Yang Mills theory. It is natural to conjecture that the $(5+1)$-dimensional NCOS theory reduces to the little string theory, in this limit.

In terms of the variables of the dual OD1 theory, the limit of the previous paragraph is $G_{\rm o(1)}\to 0$, $\tilde{\alpha}'_{\rm eff}$ held fixed. The conjecture above is thus equivalent to the assertion that the OD1 theory reduces to the IIB little string theory as $G_{\rm o(1)}$ is taken to zero at fixed $\tilde{\alpha}'_{\rm eff}$.

It is also tempting to conjecture that the OD1 theory with scale $\tilde{\alpha}'_{\rm eff}$ and parameter $G_{\rm o(1)}$ may be identified with the $(5+1)$-dimensional NCOS theory with scale $\alpha'_{\rm eff} = \tilde{\alpha}'_{\rm eff}/G^2_{\rm o(1)}$ and coupling $G_o = G_{\rm o(1)}$. Since the OD1 theory is S-dual to the $5+1$ NCOS, this conjecture amounts to a self-duality conjecture for the $(5+1)$-dimensional NCOS theory. The two theories have the same symmetries and reduce to Yang-Mills theory at low energies. We have no further evidence for this conjecture.

6.3 $p = 2$

Since we are in IIA theory now, we can lift the OD2 limit to M-theory. The scaling limit then involves an M5-brane on a point on the transverse M-theory circle, in the presence of a near critical C_{012} potential. The Planck length, invariant length of the 11-th circle and the C-field are given, in terms of OD2 parameters, by

$$R_{11} = \epsilon^{1/2} G^2_{\rm o(2)} \sqrt{\tilde{\alpha}'_{\rm eff}}\,, \qquad M_p^3 = \frac{1}{\epsilon G^2_{\rm o(2)} \tilde{\alpha}'^{3/2}_{\rm eff}}\,,$$
$$\epsilon^{012} C_{012} = \frac{1}{4\pi^2} M_p^3 \left(1 - \frac{\epsilon}{2}\right). \qquad (6.8)$$

Comparing with table 1, we find that the OD2 theory is identical to OM theory with $M^3_{\rm eff} = \frac{1}{2G^2_{\rm o(2)}\tilde{\alpha}'^{3/2}_{\rm eff}}$, on a transverse circle, of coordinate length $R = G^2_{\rm o(2)}\sqrt{\tilde{\alpha}'_{\rm eff}}$.

Notice that the tension of a fundamental string in this theory is given by $\frac{1}{2\pi} M^3_{\rm eff} R = \frac{1}{4\pi\tilde{\alpha}'_{\rm eff}}$, confirming the interpretation of $\tilde{\alpha}'_{\rm eff}$ as a scale that sets the tension of fundamental strings in ODp theories.

As a consistency check on some of the dualities described in this paper, we will relate the $(5+1)$-dimensional NCOS theory (with parameters $\alpha'_{\rm eff}$ and G_0), compactified on a circle of coordinate radius R, with a theory on a circle or radius $1/R$ through two different duality chains.

(a) By T-duality. As described in section 3, this leads to the $(4+1)$-dimensional NCOS theory with scale $\alpha'_{\rm eff}$, coupling $G'^2_o = G^2_o\sqrt{\alpha'_{\rm eff}}/R$ and radius $\alpha'_{\rm eff}/R$.

(b) By performing an S-duality, to the OD1 theory, with parameters $\tilde{\alpha}'_{\rm eff} = \alpha'_{\rm eff}G^2_o$, $G^2_{\rm o(1)} = 1/G^2_o$ and coordinate radius R. Then performing a T-duality, to the OD2 theory with parameters $\tilde{\alpha}'_{\rm eff} = \alpha'_{\rm eff}G^2_o$, $G^2_{\rm o(2)} = \frac{\sqrt{\alpha'_{\rm eff}}}{G_o R}$ on a circle of radius $\alpha'_{\rm eff}G^2_o/R$. As argued above, this is ॐ theory, with $M^3_{\rm eff} = \frac{R}{2G^2_o\alpha'^2_{\rm eff}}$, with a transverse circle of coordinate length $\alpha'_{\rm eff}/R$, compactified, in an "electric" direction, on a circle of length $\alpha'_{\rm eff}G^2_o/R$. However, using the formulae of section 4, OM theory with these parameters on an electric circle is identical to the $(4+1)$-dimensional NCOS theory, with effective scale $1/\alpha'_{\rm eff}$ and coupling constant $G_o'^2 = G_o^2\sqrt{\alpha'_{\rm eff}}/R_2$, on a circle of transverse size $\alpha'_{\rm eff}/R$, in agreement with the result of the simple T-duality described in (a) above.

6.4 $p=3$

Since here we are in the IIB theory, the OD3 theory may be analyzed by performing S-duality. From table 3 we see that the string coupling $g_{\rm str}^{(3)}$ and the RR two-form C_{45} are both of order one. After an S-duality transformation we find a D5-brane with $B_{45} = \frac{1}{2\pi G_{{\rm o}(3)}^2\tilde{\alpha}'_{\rm eff}}$ of order one, $\tilde{g}_{\rm str}^{(3)} = 1/g_{\rm str}^{(3)} = 1/G_{{\rm o}(3)}^2$ is of order one and $\alpha' = \tilde{\alpha}' g_{\rm str}^{(3)} = \epsilon^{1/2}\tilde{\alpha}'_{\rm eff}G_{{\rm o}(3)}^2$. The metric is scaled as in table 3. This is precisely the zero slope limit of [14], leading to a low energy effective NCYM (with spatial noncommutativity of rank 2). The noncommutativity parameter θ and Yang Mills coupling $g_{\rm YM}^2$ are given in terms of OD3 parameters by

$$\theta = 2\pi G_{{\rm o}(3)}^2\tilde{\alpha}'_{\rm eff}\,, \qquad g_{\rm YM}^2 = (2\pi)^3\tilde{\alpha}'_{\rm eff}\,. \tag{6.9}$$

Note that, as for $p=1$, the limit $G_{{\rm o}(3)}\to 0$, $\tilde{\alpha}'_{\rm eff}$ fixed, takes the noncommutativity parameter θ to zero (naively at least restoring Lorentz invariance) at fixed Yang Mills coupling. Once again, it is natural to conjecture that this limit leads to the IIB little string theory.

Of course, non-commutative Yang-Mills in 5+1 dimensions is nonrenormalizable, and so quantum mechanically ill defined. The OD3 theory provides a completion of 6-dimensional NCYM.

6.5 $p=4$

In M-theory the NS-5brane is an M5-brane at a point on the 11-th circle. The light 4-branes of the OD4 theory are also M5-branes — these M5-branes are wrapped on the eleventh circle intersecting the transverse M5-brane in the directions $1\cdots 4$.

The M-theory parameters that correspond to the OD4 limit are

$$R_{11} = G_{{\rm o}(4)}^2\sqrt{\tilde{\alpha}'_{\rm eff}}\,, \qquad M_p^3 = \frac{1}{\epsilon^{1/2}G_{{\rm o}(4)}^2\tilde{\alpha}'^{3/2}_{\rm eff}}\,. \tag{6.10}$$

Note that R_{11} is of order one. It is easy to check, directly in M-theory, that wrapped 5-branes of the appropriate orientation are light. $C_{01234} = \frac{1}{(2\pi)^4\epsilon G_{{\rm o}(4)}^2\tilde{\alpha}'^{5/2}_{\rm eff}}(1-\epsilon/2)$ lifts to $C_{01234,11} = \frac{1}{2\pi R_{11}}C_{01234} = \frac{1}{(2\pi)^5}M_p^6(1-\epsilon/2)$. This implies that the 5-branes wrapped on the $01234,11$ directions are light with an effective tension $T_{\rm eff}^{(5)} = \frac{1}{2(2\pi)^5}M_{\rm eff}^6 = \frac{1}{(2\pi)^5}\frac{\epsilon M_p^6}{2}$ of order one. (The dual field C_5 is of order one and hence only affects the geometry of the the $5-11$ plane by introducing a relative tilt in the coordinates.)

6.6 $p = 5$

Here we are in the IIB theory. The theory has the full SO$(5,1)$ 6-dimensional Lorentz invariance with $g_{\mu\nu} = \eta_{\mu\nu}$. As we see from table 3, the string coupling diverges like $\epsilon^{-1/2}$. We are therefore tempted to use S-duality. This converts the NS5-branes to D5-branes with a slope of order one. Superficially, this is the scaling of the little string theory [29]. But since the zero form C is of order one, the string coupling after the S-duality transformation does not go to zero, but continues to diverges like $\epsilon^{-1/2}$. So we end up with D5-branes with C and α' of order one but with a divergent string coupling.

We conclude that this theory is not weakly coupled either before or after the S-duality transformation. This situation has already been encountered in [29], where the description of D5-brane excitations of the little string theory was shown to be difficult.

7. NCOS theories at strong coupling in various dimensions

The NCOS theories constructed in [3, 2] are open string theories without a closed string sector. String perturbation theory provides an effective description of these theories only at weak open string coupling G_o. It is natural to ask if the NCOS theories have a dual description that is weakly coupled at large G_o.

Indeed, it has been argued in [3] that the strongly coupled $(3+1)$-dimensional NCOS theory is dual to a weakly coupled spatially non-commutative theory. Further, in section 3 of this paper we have argued that the strongly coupled $(4+1)$-dimensional NCOS theory is well described by 6-dimensional ॐ theory. In this section we will examine the strong coupling behavior of the NCOS theories in 6 or lower dimensions. We will also present a test of the conjectured dual description [3] of the $(3+1)$-dimensional NCOS theory. Our conjectures are summarized in table 4 below.

Dimension	NCOS theory at strong coupling
$1+1$	U(n) theory with single unit of electric flux
$2+1$	SO(8) invariant M2-brane theory
$3+1$	spatially non-commutative Yang Mills Theory
$4+1$	ॐ theory
$5+1$	self Dual

Table 4

7.1 $d = 1+1$

Consider an infinite D-string in IIB theory with background metric, and closed string coupling as in table 2.[8] The allowed values of the electric field on the D-string are quantized, and are given by (see for instance [3, eq. (2.4)])

$$\frac{2\pi\alpha'\epsilon^{01}F_{01}}{\sqrt{1+(2\pi\alpha')^2F^2}} = g_{\rm str}n\,. \tag{7.1}$$

(7.1) may be rewritten in terms of the quantities defined in section 3.1 as

$$1-\frac{\epsilon}{2} = \frac{ng_{\rm str}}{\sqrt{1+(ng_{\rm str})^2}}\,. \tag{7.2}$$

In the limit $ng_{\rm str} \gg 1$ (the near critical limit)

$$\epsilon = \frac{1}{(ng_{\rm str})^2}\,, \quad \text{i.e.} \quad {\alpha'}_{\rm eff} = \alpha'(ng_{\rm str})^2\,. \tag{7.3}$$

From table 2 we find

$$G_o^2 = \frac{1}{n}\,. \tag{7.4}$$

Thus we take the NCOS limit

$$g_{\rm str} \longrightarrow \infty\,, \qquad \alpha' = \frac{{\alpha'}_{\rm eff}}{n^2 g_{\rm str}^2}\,, \quad n, {\alpha'}_{\rm eff} \text{ fixed} \tag{7.5}$$

to obtain the $(1+1)$-dimensional NCOS theory with coupling constant $G_0^2 = 1/n$ and string tension ${\alpha'}_{\rm eff}$. Notice that G_o takes discrete values, and is bounded from above. Thus, unlike its higher-dimensional counterparts, the $(1+1)$-dimensional NCOS theory is characterized by a discrete (rather than continuous) parameter n, apart from a scale. Further, the $(1+1)$-dimensional NCOS theory cannot be taken to strong coupling.

In order to obtain a dual description of this NCOS theory, we S dualize the background described above. We find a theory of n D-strings with a single unit of electric flux, in a spacetime with

$$g'_{\mu\nu} = \frac{\eta_{\mu\nu}}{g_{\rm str}}\,, \qquad g'_{\rm str} = \frac{1}{g_{\rm str}}\,. \tag{7.6}$$

An electric field of the form $F_{01}\mathrm{I}$ (I is the identity matrix) on the D-strings is governed by the Born Infeld action for a U(1) field, times an extra factor of n from the overall trace. In this picture, the value of δ corresponding to a single unit of flux may thus be determined from an equation analogous to (7.2),

$$1-\frac{\epsilon}{2} = \frac{g'_{\rm str}/n}{\sqrt{1+(g'_{\rm str}/n)^2}} = \frac{1}{\sqrt{1+(ng_{\rm str})^2}}\,. \tag{7.7}$$

[8]This case was discussed in [20] as well as [19]. The results of this section were developed partly in discussions with I. Klebanov, L. Susskind and N. Toumbas.

In the limit of interest $ng_{\mathrm{str}}\to\infty$, so $\epsilon/2\to 1$ and the background electric field is very far from criticality. Consequently, the open string coupling and metric are equal to the corresponding closed string quantities and ${\alpha'}_{\mathrm{eff}} = \alpha'$. As $\alpha'\to 0$, both open string oscillators and gravity decouple, and the D-string world volume theory is $(1+1)$-dimensional $\mathrm{U}(n)$ Yang Mills, with a single unit of electric flux, governed by the action

$$\begin{aligned} S &= \frac{\pi\alpha'}{2g'_{\mathrm{str}}}\int d^2x\sqrt{-g'}\,\mathrm{Tr}\left(g'^{\mu\alpha}g'^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\right) \\ &= \frac{\pi g_{\mathrm{str}}^2\alpha'}{2}\int d^2x\,\mathrm{Tr}\left(\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\right). \end{aligned} \tag{7.8}$$

From (7.5), the Yang Mills coupling is

$$g_{\mathrm{YM}}^2 = \frac{1}{2\pi g_{\mathrm{str}}^2\alpha'} = \frac{n^2}{2\pi{\alpha'}_{\mathrm{eff}}} \tag{7.9}$$

and remains fixed in the scaling limit. Note that this duality predicts that the $(1+1)$-dimensional NCOS theory at the strongest allowed value of open string coupling $G_o^2 = 1$ is dual to a $\mathrm{U}(1)$ gauge theory, and so is secretly a free theory.

In summary, 1+1 $\mathrm{U}(n)$ Yang Mills with a single unit of electric flux, and gauge coupling g_{YM}^2 has a dual description as a weakly coupled $(1+1)$-dimensional NCOS theory with open string coupling $G_0^2 = 1/n$ and effective scale ${\alpha'}_{\mathrm{eff}} = \frac{n^2}{2\pi g_{\mathrm{YM}}^2}$! In the rest of this section we will use this duality to study $(1+1)$-dimensional large n $\mathrm{U}(n)$ gauge theory (with a single unit of flux) at various energies.

As shown in [30], the $(1+1)$-dimensional $\mathrm{U}(n)$ theory with a single unit of electric flux reduces to a free $\mathrm{U}(1)$ theory at low energies, as the $\mathrm{SU}(n)$ part of this theory is massive. The dual weakly coupled NCOS theory also reduces to a free $\mathrm{U}(1)$ theory at low energies; indeed it may be used to predict that the mass gap of the $\mathrm{SU}(n)$ part of the $\mathrm{U}(n)$ theory is $\frac{1}{\sqrt{{\alpha'}_{\mathrm{eff}}}} = \sqrt{2\pi g_{\mathrm{YM}}^2/n^2}$.

For a range of energies above the mass gap, and at large n, the duality derived in this section predicts that the weakly coupled degrees of freedom of the gauge theory are open strings, with a tension

$$\frac{1}{4\pi{\alpha'}_{\mathrm{eff}}} = \frac{g_{\mathrm{YM}}^2}{2n^2} \tag{7.10}$$

and effective coupling $G_o^2 = 1/n$. Indeed these stringlike excitations are easily identified in the gauge theory. First recall why the $\mathrm{SU}(n)$ part of the theory is gapped. Excitations of the $\mathrm{SU}(n)$ theory involve excitations of the scalars X^I, i.e. configurations involving separated D1-branes, as the gauge field has no dynamics. However, because of the background electric field, all such excitations cost energy. For instance, a configuration in which a single D-string is taken to infinity (the $\mathrm{SU}(n)$ is

Higgsed to $\mathrm{SU}(n-1)\times \mathrm{U}(1)$), has energy per unit length above that of the vacuum given by the BPS formula[9]

$$\frac{g^2_{\rm YM}}{2n} - \frac{g^2_{\rm YM}}{2(n-1)} \approx \frac{g^2_{\rm YM}}{2n^2} \quad (n \gg 1) \tag{7.11}$$

as, in this limit, the electric flux is shared by $n-1$ rather than n D-strings. But this tension agrees exactly with that of the NCOS string. Thus an NCOS open string of length L is identified with a configurations of the n coincident D-branes of the gauge theory, in which the background electric flux is shared between $n-1$ of the D-strings over a length L. The last remaining D-string is free to fluctuate in the R^8 transverse to the D-branes over this segment of length L, but is bound to the branes everywhere else, resulting in an open string of length L with Dirichlet boundary conditions.

As with all string theories, the effective coupling of the NCOS theory grows with energy, and at energies much higher than the mass gap, the NCOS strings, (and, therefore, the "flux" strings of the gauge theory, described in the previous paragraph) are strongly coupled. Indeed, at squared energies much larger than $g^2_{\rm YM}n$, the usual W-bosons of the gauge theory constitute the weakly coupled variables for the gauge theory.

7.2 $d = 2+1$

IIA theory in the NCOS limit of table 2 may equivalently be described as M-theory on a circle of proper radius $R_{11} = g_{\rm str}\sqrt{\alpha'} = G_o^2\sqrt{\alpha'_{\rm eff}}$ and a Planck mass $M_{\rm p}$ that goes to infinity $M_{\rm p}^{-1} = g_{\rm str}^{1/3}\sqrt{\alpha'} = G_o^{2/3}\alpha'^{1/3}\alpha'^{1/6}_{\rm eff}$. Recall that a D2-brane in IIA theory, with no F flux on its worldvolume, maps to an M2-brane at a point on the 11-th circle. The dynamics of the gauge field on such a 2-brane maps to the dynamics of the compact scalar (representing the position of the M2-brane in the 11-th direction) on the M2-brane world volume.

Now consider a D2-brane with a large electric field, as in table 2. One may choose to regard $F_{\mu\nu}$ instead of A_μ as the dynamical variable in the Born Infeld action on the D2-brane, if one simultaneously introduces a Lagrange Multiplier field ϕ that enforces the constraint $dF = 0$;

$$\begin{aligned} S = {} & \frac{1}{(2\pi)^2 g_{\rm str}\alpha'^{3/2}} \int d^3x \sqrt{-\det\left(g_{MN} + 2\pi\alpha' F_{MN}\right)} + \\ & + \frac{1}{2}\int d^3x \sqrt{-g}\,\epsilon^{MNP}\partial_M\phi F_{NP}\,. \end{aligned} \tag{7.12}$$

The equation of motion that results from varying this action with respect to F_{MN}, specialized to the case of a diagonal metric g_{MN} and a constant background electric

[9] A simple way to check the factors in this formula is to recall that the tension of a $(p,1)$ string is $\frac{1}{2\pi}\sqrt{p^2/g^2_{\rm str}+1}$ and that in 1+1 dimensions $g^2_{\rm YM} = \frac{g_{\rm str}}{2\pi\alpha'}$.

field F_{01}, is

$$\partial_2\phi = \frac{1}{4\pi^2 g_{\rm str}\alpha'^{3/2}} \frac{(2\pi\alpha')^2\epsilon_2^{01}F_{01}}{\sqrt{1+g^{11}g^{00}(2\pi\alpha' F_{01})^2}}\,. \qquad (7.13)$$

The scalar field ϕ in (7.13) is dimensionless, and is compact of unit periodicity[10] $\phi \equiv \phi + 1$. In the NCOS limit of table 2, (7.13) may be rewritten as

$$\partial_2\phi = \frac{1}{2\pi g_{\rm str}\sqrt{\alpha'}} = \frac{1}{2\pi R_{11}}\,. \qquad (7.14)$$

$2\pi R_{11}\phi = X^{11}$ is identified geometrically with the position of the brane in the 11-th direction. (7.14) implies that in the presence of a near critical electric field, the M2-brane tilts at an angle of 45 degrees in the 2-11 coordinate plane

$$X^{11} = x^2\,. \qquad (7.15)$$

However, in the NCOS limit, $g_{22}\to 0$, so that, when angles are measured in terms of physical distances, the M2-brane is oriented almost entirely in the $0, 1, 11$ directions. More precisely, the 2-dimensional NCOS scaling limit has a dual description in terms of an M2-brane extended in the $0, 1,$ directions, and spiraling around the 2-11 cylinder.[11] Successive turns of the spiral are separated by physical distance

$$\Delta X \approx 2\pi\sqrt{g_{22}}R_{11} = 2\pi G_o^2\sqrt{\alpha'}\,. \qquad (7.16)$$

The field ψ on the worldvolume of the M2-brane, normalized so that its kinetic term is $\frac{1}{2}\int d^3x(\partial\psi)^2$, is related to the physical displacement X by $\psi = (2\pi)^{-1}M_{\rm p}^{3/2}X$. Combining this with (7.16) we find that the spacing between successive turns of the spiral in ψ space is given by

$$\Delta\psi = \frac{G_o}{\alpha'^{1/4}_{\rm eff}} \qquad (7.17)$$

and is finite in the NCOS limit.

Interactions between successive windings may be ignored only for energies $\omega \ll G_o^2/\sqrt{\alpha'_{\rm eff}}$. For $G_o \ll 1$ we thus obtain a complicated interacting theory at energy scale $\sim 1/\sqrt{\alpha'_{\rm eff}}$. On the other hand, in the strong coupling limit $G_o\to\infty$, interactions may be ignored at all energies, and the NCOS theory reduces to the free SO(8) invariant theory of a single M2-brane.

8. Moduli counting

Since this paper involves branes in $\mathcal{R}^n$ in constant background fields, we would like to make a few comments about such backgrounds. We start by considering Dp-branes in $\mathcal{R}^{10}$. In the absence of the branes a constant NS B-field can be gauged

[10]The periodicity of ϕ may be deduced as follows. Consider the theory on a circle in the 2 direction. The r.h.s. of (7.13), integrated over the circle, is an integer, by the 3-dimensional analogue of (7.1). Hence $\oint dx^2\partial_2\phi = n$.

[11]This picture was also developed by O. Aharony and C. Vafa (private communication).

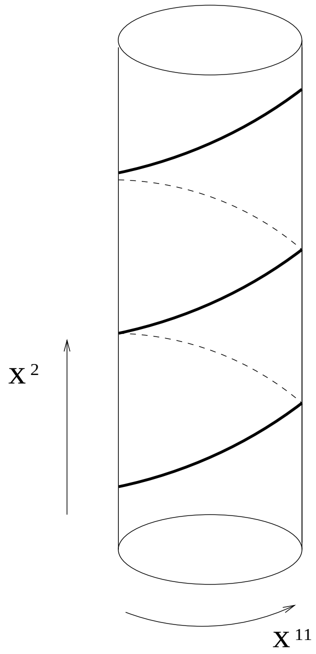

Figure 1: When lifted to M-theory, a D2-brane in the $0, 1, 2$ directions with a near critical F_{01} field is an M2-brane that that fills the 0-1 plane and spirals round the 2-11 cylinder. The pitch of the spiral is related to the open string coupling G_o and goes to infinity at strong coupling.

away. When the branes are present such a gauge transformation changes the value of the field strength F at infinity $F(\infty)$, since only $F + B$ is gauge invariant. We can either fix the boundary conditions $F(\infty)$, and then B is meaningful, or gauge transform B to zero and focus on the background $F(\infty)$, or more generally discuss the gauge invariant background $F(\infty) + B$. The situation with M5-branes in $\mathcal{R}^{11}$ is somewhat more interesting. As for D-branes, only $\mathcal{H}(\infty) = H(\infty) + C$ is gauge invariant. However, here there is a new element because the field strength $\mathcal{H}$ is a constrained field. For a single M5-brane and a small and slowly varying $\mathcal{H}$ it has to be selfdual, and for generic values of $\mathcal{H}$, which are still slowly varying the condition is more complicated [22]. This equation should also be imposed on the boundary values $\mathcal{H}(\infty)$ which characterize the problem [14]. This means that the problem of M5-branes in flat $\mathcal{R}^{11}$ depends on $(6 \times 5 \times 4)/(2 \times 3!) = 10$ parameters, where the factor of 2 arises from this generalized self-duality condition.

We now follow these parameters as the M5-branes are compactified on a circle to become D4-branes in type-IIA string theory. For simplicity we consider a single M5-brane. The 6-dimensional three-form H-field leads to a two-form field strength $F = \oint H$ in five dimensions and a three form. The M5-brane equation relates them and determines one of them in terms of the other [31]. Hence, we can take the degrees of freedom to be ordinary gauge fields with field strength F.

For slowly varying fields the dynamics of the D4-brane is well known to be controlled by the lagrangian

$$L = h(F + B) + C \wedge (F + B)\,, \tag{8.1}$$

where h is the DBI lagrangian (for a review see [32]). The term proportional to the RR field C can be dropped when C is a constant since it is a total derivative. (8.1) is invariant under the electric gauge symmetries

$$\delta A = d\lambda_e + \Lambda_e\,, \qquad \delta B = -\Lambda_e\,. \tag{8.2}$$

We note that for constant B and C we are free to specify 20 independent parameters as well as the boundary conditions $F(\infty)$. However, only $F(\infty)+B$ is gauge invariant and meaningful, and the terms proportional to C do not affect the local dynamics. Therefore the problem is characterized only by 10 parameters, exactly as for its ancestor M5-brane.

Let us perform a duality transformation on the lagrangian (8.1). We do that by viewing F as an independent field and by introducing a Lagrange multiplier V to implement the Bianchi identity for F. The lagrangian L is replaced by

$$h(F + B) + C \wedge (F + B) - V \wedge dF\,. \tag{8.3}$$

Next, we integrate by parts to replace (8.3) by

$$h(F + B) + (C + dV) \wedge (F + B)\,. \tag{8.4}$$

The lagrangian (8.4) is invariant under the magnetic gauge symmetries

$$\delta V = d\lambda_m + \Lambda_m\,, \qquad \delta C = -\Lambda_m\,. \tag{8.5}$$

The equation of motion of F is algebraic

$$h'(F + B) + C + F_D = 0\,; \quad F_D = dV\,. \tag{8.6}$$

It has a number of consequences:

(1) The two field strengths F and F_D are the two form and three form which are obtained by dimensional reduction of the M5-brane field H, and B and C are the dimensional reduction of the higher-dimensional C-field. Here we see explicitly how they are related. As shown in [31], equation (8.6) is the dimensional reduction of the generalized self-duality equation of [22].

(2) Equation (8.6) can be solved $F + B = f(C + F_D)$ and F can be integrated out to express the the theory in terms of the dual variables as

$$L_{\text{dual}} = h(f(C + F_D)) + (C + F_D) \wedge f(C + F_D)\,. \tag{8.7}$$

We see that the dual lagrangian is independent of B and that the dependence on the background RR field C is non linear.

(3) In (8.1) the constants C are arbitrary, as they multiply total derivative terms. However, they do affect the boundary conditions of the dual variables V. To see that, we should examine (8.6) at infinity

$$h'(F(\infty) + B) + C + F_D(\infty) = 0\,. \tag{8.8}$$

Clearly, the value of $F_D(\infty)$ depends on C. As for the electric variables, only $C + F_D(\infty)$ is gauge invariant and physical. Furthermore, these constants are determined in terms of the electric constants $F(\infty)+B$. Therefore, the problem is characterized by $(5 \times 4)/2 = 10$ parameters, exactly as for the M5-brane we started with.

It is straightforward to repeat this analysis for D3-branes in $\mathcal{R}^{10}$. Here the background depends on $(4 \times 3)/2 = 6$ parameters from the NS B-field, which can be interpreted as boundary values of the field strength F at infinity. The $(4 \times 3)/2 = 6$ parameters in the RR C-field multiply total derivative terms and do not affect the dynamics. Hence, the total number of independent gauge invariant two-form parameters is six. S-duality transformation can be performed as above except that now F_D and C are two forms. Again, equation (8.8) relates the boundary values of F and F_D in terms of the parameters B and C.

A natural gauge choice is $B = C = 0$, and then the boundary conditions on the dynamical fields are $h'(F(\infty)) + {}^*F_D(\infty) = 0$. With this choice S-duality exchanges non-zero background electric field with background magnetic field. Another natural choice is such that the dynamical fields vanish at infinity $F(\infty) = F_D(\infty) = 0$. Then the NS and the RR fields are related by $h'(B) + {}^*C = 0$. Here S-duality exchanges the related values of B and C. The local dynamics depends only on the value of the NS B-field, so that S-duality can be described as mapping $B \to -{}^*h'(B)$ (it is straightforward to check that this transformation is a Z_2 transformation).

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